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LETTER TO THE EDITOR

Time-non-local conserved currents for gauge theories

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Abstract. Manifestly conserved but time-non-local currents are exhibited for source-free Yang–Mills and gravity theories in arbitrary dimensions. They are just the familiar spinorial currents of the associated supersymmetric models, properly reinterpreted, and are infinite in number. The underlying hidden symmetry is supersymmetrisability.

It has long been suspected that source-free non-abelian gauge theories possess a hidden symmetry to which is associated an infinite set of spatially non-local conserved currents, and that these may imply integrability of the models (see, for example, Polyakov 1979). We point out here that time-non-local conserved currents at least are already 'known' for an even wider class of theories. In any bosonic system which is supersymmetrisable—this is the hidden symmetry—the familiar supercurrents of the corresponding supersymmetric models, when properly reinterpreted, are conserved. Whether they are also useful is not yet clear.

We shall deal here with the gauge theories, for concreteness. For Yang–Mills, our main result is the obvious statement that the supercurrent is conserved,

$$\partial_\mu J^\mu \equiv \partial_\mu \operatorname{Tr} F\cdot\sigma\gamma^\mu\lambda = 0 \tag{1a}$$

by virtue of the equations

$$D_\mu F^{\mu\nu} = 0, \qquad \not{D}\lambda = 0. \tag{1b}$$

Here D_μ and $F^{\mu\nu}$ are the usual Yang–Mills covariant derivative and field strength. The λ are spinors in the adjoint representation of the (arbitrary) algebra, to be regarded as non-local functionals $\lambda[A]$ of the gauge field, *not* as independent dynamical objects. In particular, as is clear from (1*b*), they are *not* sources of the field. If they are taken to transform as gauge vectors, then the current J^μ is gauge invariant. In gravity, the immediate analogue of the previous construction would lead to a gauge (coordinate)-variant current, because the associated supersymmetry is local and because a genuine vector–spinor current would have to be covariantly rather than ordinarily conserved. Consider a vector–spinor field $\psi_\alpha[G]$ obeying the covariant Rarita–Schwinger equation $R^\mu = 0$. It is a non-local functional of the torsion-free geometry G satisfying the source-free Einstein equation $G_{\mu\nu} = 0$; hence the consistency conditions $D_\mu R^\mu = 0$ are fulfilled. (Inconsistency of Yang–Mills coupling rules out currents constructed from spin-$\frac{3}{2}$ solutions there.) Now divide R^μ into a part $R^\mu_{\rm L}$ depending only on the flat metric, plus a remainder $R^\mu_{\rm N}$. The former is identically

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conserved, $\partial_\mu R^\mu_L \equiv 0$; consequently (since $R^\mu_L + R^\mu_N = 0$), the conserved current is simply R^μ_N. This current is clearly global Lorentz-covariant, but not coordinate covariant; nevertheless it is conserved in any frame.

We conclude with some comments, conjectures and questions. (A somewhat unrelated, but tempting, conjecture is that self-duality of finite action solutions in Euclidean four-space can also be decided by using this hidden symmetry, given the close relation between duality and helicity in supersymmetry.) The above results are of course independent of the supersymmetric scaffolding, and provide further examples of the constraints implied by this hidden symmetry even when supersymmetry is not physically realised. They hold for arbitrary dimensions (for $D = 4$, $\gamma \cdot J = 0$ as well) and signature, and at the classical or quantum level, since there are no supercurrent anomalies in the source-free theories under consideration. Note also that since D_μ is not renormalised, neither are λ or J^μ. When arbitrary sources are present, the hidden symmetry is lost and the supercurrents are no longer conserved, except of course for the supersymmetric theory where the fermions are dynamical variables and (1*a*) is satisfied with a λ-current source of the gauge field equations. (For the supersymmetric theory J^μ is the (time-local) Noether current of a true, rather than hidden, (super)symmetry, and there is only one, rather than an infinite set, per field configuration.) We have not considered here the currents associated with extended supersymmetry, which may also yield (by appropriate truncations) further information when genuine (but supersymmetrically special) sources are present. The insight provided by supersymmetry may also be expressed as follows. For a spinorial zero mode ($\not{D}\lambda = 0$), $\gamma^\mu\lambda$ is a covariantly conserved, and therefore uninteresting, colour vector. What is remarkable, and provided only by the hidden symmetry, is that there exists a quantity, $F \cdot \sigma$, which acts like a (colour) Killing vector: it contracts with $\gamma^\mu\lambda$ to form the normally conserved colour singlet J^μ. (This construction is non-trivial: merely postulating a form Tr $F^{\mu\nu}\phi_\nu$, say, for the current implies very strong constraints on ϕ^ν, namely that Tr $F^{\mu\nu}(D_\mu\phi_\nu - D_\nu\phi_\mu) = 0$. With the solution $\phi_\mu = D_\mu\alpha$, the current is seen to have the useless identically conserved form ∂_ν Tr$(F^{\mu\nu}\alpha)$.) In the abelian limit, the currents become trivial: they are just linear in the gauge field, since λ reduces to a field-independent solution of the free Dirac equation. It would be interesting to relate these ideas to another property of supersymmetrisable bosonic actions, namely that they can be transformed into free form by time-non-local field redefinitions (Nicolai 1980, 1981, Deser and Nicolai 1981a, b).

We have stated that there are infinitely many currents J^μ. More precisely, their number is that of the independent zero modes λ, $\not{D}\lambda = 0$, in a given on-shell gauge field configuration. From a perturbative point of view, there is (at least in Minkowski signature) a complete set of solutions $\lambda[\lambda_0: A_\mu]$ parametrised by the free 'in' states λ_0, $\not{\partial}\lambda_0 = 0$, or equivalently by arbitrary initial time choices $\lambda(t = 0)$. The λ may be taken to be commuting or anticommuting c numbers since they are not dynamical variables; the commutation properties of two λ are governed essentially by their gauge field dependence. The λ are also formally related to the time-non-local analogues of the usual string variables. The asymptotic behaviour of the currents will depend on that of the λ; for sufficiently localised $\lambda(t = 0)$, the corresponding conserved spinorial charges should be well defined. Other conserved time-non-local currents are quantities such as Tr$\bar{\lambda}_1\gamma^\mu\lambda_2$ (or $\varepsilon^{\mu\nu\alpha\beta}\bar{\psi}_{1\nu}\gamma_\alpha\gamma_5\psi_{2\beta}$ for gravity) where (λ_1, λ_2) are any two solutions of the covariant Dirac equation. However, they do not give rise to interesting charges, since the latter depend only on the arbitrary initial parameter choices $\lambda_i(t = 0)$ rather than on the gauge fields. This is not the case for the supercharge, which does depend explicitly on $F_{\alpha\beta}(t=0)$ as well. Nor is this charge just a 'Hamilton–Jacobi quantity, i.e. it is *not* defined to be $\int J^0(t=0)\mathrm{d}^3x$ for all times. Its time non-locality, i.e. explicit dependence on time, means however that although conserved it does not commute with the Hamiltonian.

The major question raised by these considerations is the one motivating the original searches—do these currents imply integrability of the gauge field equations? Dealing with their time non-locality may prove even more difficult.

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